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ABSTRACT

Guidelines that were prepared for use in curriculum development efforts by schools and districts throughout the state of Washington are presented in this document. The philosophy of mathematics education reflected in these guidelines is presented, as are a discussion of areas to be addressed if needed improvements in the K-8 mathematics program are to be made, and material related to six mathematics content strands. These strands are: (1) problem-solving; (2) measurement; (3) geometry; (4) number properties, theory, and computation; (5) probability and statistics; and (6) algebra. Each strand consists of introductory comments (which provide information on scope and sequence, the relative importance of instructional emphasis, and classroom implementation concerns) and a listing of learning results (objectives) and their corresponding instructional implications. These results and implications are organized by grade levels (grades K-3, grades 4-6, and grades 7-8) within each strand. A bibliography of selected instructional resources, compiled from materials that the curriculum writing team found useful in their own classroom experiences, is provided at the end of each set of learning results and instructional implications. (JN)

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GUIDELINES FOR K-8 MATHEMATICS CURRICULUM

Toward Meeting Present and Future Needs

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GUIDELINES FOR K-8 MATHEMATICS CURRICULUM

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May, 1984

SPI

DR. FRANK B. BROUILLET Superintendent of Public Instruction

MESSAGE FROM THE SUPERINTENDENT OF PUBLIC INSTRUCTION

As society moves toward the twenty-first century there will be a need for all citizens to become prepared for an increasingly technological age. Certainly our schools must point the preparation of our K-12 students in this direction. To this end the State Board of Education adopted in May 1983, increased graduation requirements and requested my agency to prepare program suggestions and curriculum guidelines to match the graduation requirements. The State Board also stipulated that the guidelines should reflect the desire to achieve excellence across both academic and vocational areas, and to prepare students with the skills required for college and work.

It is my hope that school districts will find these mathematics guidelines, the first in the series of curriculum guidelines to be developed, helpful as they upgrade curriculum, revise and complete Student Learning Objectives. and engage in other program improvement efforts.

As we proceed then with joint efforts to upgrade mathematics curriculum, I strongly urge that you adopt a dual thrust of equity and excellence which will -

- facilitate improvement and change in mathematics curricula
- enhance the quality of teaching in these fields
- increase the enrollment of students, particularly women and minority males, in mathematics.

The implementation of a plan to develop the Mathematics Curriculum Guidelines resulted in the involvement of many educators from all levels throughout the state. Many hours have been given in order to reach consensus on these guidelines. I congratulate the educators whose names appear on the pages of this document for their excellent work in developing a set of guidelines for K-8 mathematics curriculum. The ideas and concepts expressed herein will truly lead us forward in meeting present and future needs in mathematics education.

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A MESSAGE TO THE READER -

The ultimate goal was to prepare guidelines to be used in curriculum development efforts by schools and districts throughout the state. The resulting document draws heavily on recent research, reports, and studies on the current state of mathematics education and the need for changes in content, emphasis, and methodology in order to meet the challenges of this decade and beyond.

In order of priority, it is strongly encouraged attention be given to:

- The Philosophy and Implementation.

  It is critical that you understand and accept the thrust of this document and the necessity for effective implementation if it is to serve its intended purpose. If your interest is limited to an overview of the nature and implementation requirements of a mathematics program that will meet present and future needs, this is the section to which you should give your attention.

- The introductory statement for each content strand.

  Each statement provides, in general terms, information concerning scope and sequence, relative importance of instructional emphasis, and classroom implementation concerns.

- The Learning Results and Instructional Implications within each content strand.

  These are organized by grade-level bands (K-3, 4-6, 7-8) within each strand and are specifically intended for use for curriculum development once the decision has been made by schools/districts to assume this task. As an addendum to each set of Learning Results and Instructional Implications, a listing of instructional resources is included. It should be understood that these listings were compiled from materials that members of the Core Writing Group have found useful in their own classroom experiences. Some of the lists are lengthy; but there is the assumption that those involved in curriculum development and implementation will be selective in their choices of resources from among those listed and will perhaps replace some of these with others on the basis of their own knowledge and experiences.

There may be a concern that the Instructional Implication statements that begin, "The teacher should . . ." be read and understood within the context of the other sections of the document. It is both unfair and unrealistic to place the major burden of effecting the needed changes on the already overloaded classroom teacher. Responsibilities must be shared by the funding agencies, administrators at the local, district, regional and state levels, teacher training institutions, curriculum developers, and classroom teachers. There is also an obvious need for the publishing community to produce textbooks and supporting materials that respond to the changes presented in this document.

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PHILOSOPHY

"Developing thoughtful behavior in mathematics"[1] is an expression that identifies the thrust of this document. Within this context, problem solving assumes first priority on a continuing basis throughout the mathematics education of students.[2]

So that there is no misunderstanding, problem-solving activities are those that demand much more than just the application of previously learned skills in familiar situations as is often the case with standard textbook "word problems." Solving a problem requires the use of understandings and skills in unexpected, unplanned, and often non-routine situations. Problem-solving activities demand that a broad range of strategies be learned and applied in a variety of settings at appropriate levels of sophistication.

The process of developing thoughtful behavior in mathematics requires that throughout the grades and within each strand, instruction should progress from the concrete level through pictorial representation, and only then to abstract symbolization. Teachers should use their knowledge of the cognitive development of students in order to recognize when a student needs to move from one level to another. Matters pertaining to learning styles should also be taken into consideration in the design of learning activities.

The foregoing, together with the following considerations, set the direction for the improvements needed in K-8 mathematics programs in schools and districts throughout the state.

- Mathematics educators[3] must make a strong effort to counter the restrictive influence of the "basic skills" push of recent years. Since the term has become a part of the popular lexicon, we can expect it to remain with us in the foreseeable future. For this reason, we must promote an understanding of the term in the broader sense of this document: i.e., every strand is an integral part of the basic skills package.[4]

- Teachers must be aware of the fact that their own attitudes toward mathematics affect their students' attitudes. Teachers should communicate enthusiasm for the subject and should believe that all students can achieve success. Students will acquire a positive and friendly attitude toward mathematics if they feel free to risk being "wrong," are given time to explore and discover, are provided with learning experiences that are within their success range, and experience applications of mathematics in "real life" situations.

- Students should use calculators to perform multi-digit computations, thus reducing the time given to pencil/paper drill. Calculators should also be used in concept development and in the application of mathematics to "real life" situations.

- Teachers must place increased emphasis on developing students' abilities in estimation, approximation, mental arithmetic, and the assessment of the reasonableness of answers. As an important corollary, students must learn the basic number facts and attain facility in mental arithmetic in order to assure success in these areas.

- As computer software becomes increasingly available for the facilitation of concept development and the acquisition of problem-solving skills, teachers should integrate, whenever appropriate, the use of computers in the instructional program.

However, instruction in computer programming should not encroach upon the time devoted to mathematics instruction.

- With the increased emphasis on problem solving, the opportunity for enriching the mathematics program without resorting to vertical acceleration is apparent. Schools and districts which currently accelerate large numbers of students through their course work should evaluate the results of such action carefully and thoroughly to make sure that the best interests of students are served.[5]

- Mathematics achievement must be assessed by a broader range of evaluation instruments than the Student Learning Objective (SLO) mastery tests and standardized tests of current practice. Such tests focus mainly on the lower-level cognitive skills. The evaluation of problem-solving abilities and attitudinal development demands new approaches.

- Mathematics educators must give attention to the reasons for the underrepresentation of females and some minorities in higher-level mathematics courses. While the problem surfaces at the high school level, it is a K-12 concern and must be treated as such. Districts/schools should affiliate with or otherwise make use of opportunities provided by networks dedicated to overcoming such underrepresentation in attacking the problem at the district/school level.[6]

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[1]Developing Thoughtful Behavior in Mathematics was the title of a recent publication of the Washington State Mathematics Council. The late Robert W. Wirtz, a pioneer in the development of problem-solving activities for school mathematics, first used this phrase in a discussion of his philosophy of mathematics education.

[2]Problem solving is the first priority of An Agenda for Action, Recommendations for School Mathematics of the 1980s, a publication of the National Council of Teachers of Mathematics.

[3]"Mathematics educators" is a term that is meant to include all teachers of mathematics at every level.

[4]See the National Council of Supervisors of Mathematics position paper, The Ten Basic Skills Areas.

[5]See the position statement on "vertical acceleration" in the September, 1983 National Council of Teachers of Mathematics News Bulletin.

[6]E.g. (1) Math/Science Network, Lawrence Hall of Science, University of California;
(2) Equity Project, National Council of Teachers of Mathematics;
(3) Minorities and Mathematics Network, Chicago Associates for Social Research;
(4) Washington State Mathematics Equity Project, Superintendent of Public Instruction

IMPLEMENTATION

Positive action must be taken in each of the following areas if the needed improvements in the K-8 mathematics program are to be made.

Preservice Training -

Teacher training institutions have the responsibility to develop both content and methods courses that will support the implementation of program changes presented in this document. Special attention should be given to courses that would target the special needs of those preparing to teach at each of the three grade-band levels: K-3, 4-6, 7-8.

Inservice Training -

Comprehensive inservice programs must be funded in districts throughout the State. The need is immediate and should receive top priority. While attention to preservice training, as indicated above, will in time exert a positive impact, the fact is that the great majority of students, during this decade at least, will be taught by teachers who are currently in the classroom. Few of these teachers have had the training necessary to implement fully a mathematics program in which the primary focus is developing thoughtful behavior. While this is particularly the case with respect to the increased emphasis on problem solving, the other content strands are also in need of attention. Additionally, instructional strategies which include the use of concrete manipulatives in all of the content strands must become a part of each teachers's repertoire. The matter of learning styles and the role of cooperative learning in the mathematics classroom are also areas in which better understanding on the part of many teachers is needed. With these needs in mind, ongoing funding must be provided for intensive inservice programs in districts throughout the State. Outstanding mathematics teachers at each level, university mathematics educators, and mathematics curriculum coordinators should plan and implement such programs cooperatively. If such inservice programs are not provided, the significant improvements in mathematics education envisioned in this document will not occur except in isolated cases.

Assignment of Teachers -

Teachers assigned to teach mathematics at grades 7 and 8 should be those who specialize in mathematics. Generally this will mean that only those who have at least the equivalent of a minor in mathematics should be assigned to mathematics classrooms for these grades; however, an equally important consideration is the teachers' interest in and excitement for the subject and their ability to provide motivational learning experiences for their students. Teachers who "come late" to the realization that mathematics teaching is their forte and love are often among the most effective teachers at this level in spite of their relative lack of prior preparation. These teachers can often be recognized by their continuing efforts to upgrade their expertise, taking additional college-level course work, participating in inservice workshops, and involving themselves in professional activities by attending or otherwise participating in local, regional, and national conferences.

Much of what is appropriate for the assignment of teachers to grades 7-8 mathematics classrooms should also be considered for assignment of teachers to intermediate classrooms. When possible, "mathematics specialists" should

be assigned to teach all of the mathematics classes at these grade levels. In particular, only those teachers who enjoy teaching mathematics should be involved.

The greatest advances in mathematics education during the past few years have probably taken place in primary classrooms. This has come about through the introduction of hands-on experiential programs that are now well-established in many classrooms throughout the state. Both inservice and preservice efforts need to be intensified to bring ever greater numbers of primary teachers into programs of this kind. Again, only teachers who enjoy teaching mathematics should be involved in the instructional program at these grade levels.

Administrative Support for Involvement in Professional Activities -

A most effective means for schools/districts to upgrade the quality of mathematics instruction is to support the involvement of teachers in the activities of the various mathematics professional organizations: local, state, and national. Districts should encourage teachers to attend conferences/meetings of these organizations through the provision of release time and/or financial support. It should be noted that the publications of these organizations provide teachers with an abundance of instructional resources and useful instruction ideas; attendance at conferences/meetings provides a degree of stimulation and motivation through exposure to regionally and nationally recognized mathematics educators which is unavailable elsewhere.

Instructional Resources -

Since the textbook will undoubtedly remain a primary instructional resource in most mathematics classrooms, pressure must be brought to bare on the major publishers in order that they respond to the challenge of providing texts and teachers' guides that truly incorporate the changes in content and methodology set forth in this document. The practice of producing relatively minor revisions (whether under a new title or just a new copyright date) to which a "problem solving" label is appended will not suffice. Since we cannot afford to wait for a selection of acceptable texts to appear, provision must be made in curriculum development efforts (see below) for the development and/or identification of learning resources that will serve in their stead. In addition, mathematics teachers at all levels must have access to a wide array of instructional aids. A look at the recommended resources for each strand will make clear the extent of this need. The abundance of manipulative materials at all grade levels as well as access to calculators and computers pose funding needs which must be met; however, funding for computer software must be preceded by the establishment of effective and reliable procedures for evaluating software and for sharing the results of such evaluations. The Office of the Superintendent of Public Instruction (SPI) and/or other statewide agencies should accept the responsibility for coordinating software evaluation matters.

Curriculum Development -

Schools/districts must find the means to finance extensive curriculum development projects that will incorporate the needed changes and target the necessary instructional resources. Together with those having direct curriculum development responsibilities within schools/districts, teachers need to be involved in such projects to insure that they develop a sense of "ownership" of the

resulting program. Without this, all attempts to implement program changes will be severely handicapped, if not defeated. An additional reason for maximizing teacher involvement is that, in terms of funds expended, this is a most effective means for teachers to acquire the skills and understandings they will need for the classroom tasks they must assume and the leadership roles they may be expected to play in satisfaction of the inservice needs of the school/district.

I. K-8 PROBLEM SOLVING STRAND

Problem solving is a process that requires students to apply their mathematical and thinking skills, and often their group interaction skills, in order to reach a solution. The transfer of problem-solving skills from one situation to another and the application of such skills in decision making processes are necessary in order that individuals may deal successfully with the world around them. Students must be given plenty of time to "wrestle" with problems, to try different strategies, to analyze data, and to evaluate their solutions. Regardless of the student's level of sophistication, he/she is capable of active involvement in generating and solving problems. The following considerations should also be kept in mind.

- The Problem Solving Strand is concerned with the process of problem solving - the strategies that must be taught together with the provision of experiences for their use; however, problem solving must not be considered in isolation from the other strands. It should permeate each of them on a continuing basis throughout the students' mathematics education.

- The student should perceive the teacher as an active problem solver who enjoys working on problems, not merely getting right answers; who has the ability to look at errors as opportunities for learning instead of as unfortunate mistakes; who values persistence over speed; and who most often verifies his/her own results without reliance on an answer book or another authority.

- Problem solving offers abundant opportunities for students to work cooperatively. Problem-solving experiences are most productive in a cooperative environment in which the teacher observes the interaction, generation of ideas and strategies, and work procedures for use in later discussions.

- As will be evident, the problem-solving strategies identified for the different grade bands on the pages that follow are the same. This simply means that the strategies are universally useful regardless of the age of the students with the understanding that the selection of specific problem-solving activities must necessarily be adjusted to their level of sophistication.

I-A. GRADES K-3

Learning Results	Instructional Implications
1. Students sort, group and label collections of objects.	1.1 Teachers should provide students with opportunities to observe and identify attributes of objects in their environment.
	1.2 Teachers should provide students with opportunities to experiment with sorting, grouping, and labeling many different kinds of objects as well as with subdividing collections of one kind of object according to some well-defined plan of their own invention.

Learning Results	Instructional Implications
	1.3 Teachers should encourage students to verbalize their methods of sorting, grouping, and labeling.
	1.4 Teachers should encourage students to record their results using diagrams, charts, graphs, and/or pictures.
2. Students recognize, describe, create, reproduce, extend, predict, and translate patterns.	2.1 Teachers should provide visual, auditory, and physical experiences for pattern recognition and description.
	2.2 Teachers should provide a variety of experiences that encourage students to create, reproduce, extend, and predict patterns.
	2.3 Teachers should provide activities in which students are required to translate patterns from one medium to another.
3. Students use various strategies in problem-solving situations.	3.1 Teachers should provide an abundance of problem-solving activities leading to the understanding and use of strategies such as the following: - looking for and using patterns; - constructing tables, charts, and graphs; - using estimation skills; - making organized lists; - acting out problem situations; - guessing, testing, and evaluating; - drawing a picture or making a model; - solving a simpler or similar problem; - brainstorming; - looking for counter examples; - working backwards

Learning Results	Instructional Implications
4. Students become autonomous, confident problem solvers.	4.1 Teachers should present problems that are within the experience range of the students to assure that they become personally involved, thus leading to greater retention of the concepts and strategies for future use.
	4.2 Teachers should capitalize on problem situations from other disciplines as appropriate.
	4.3 Teachers should help students analyze the scope of the problem and understand what they are being asked to find.
	4.4 Teachers should establish an atmosphere of acceptance and capitalize on the students' imagination, intuition and organizational abilities.
	4.5 Teachers should allow ample time for students to "wrestle" with problems in order to focus attention on the thinking process rather than the solution (valuing persistence over speed).
	4.6 Teachers should provide problems including those of a non-routine nature at a level of sophistication that presents a challenge but which is not unduly frustrating.

Bibliography of Selected Instructional Resources

Mathematics Their Way, Mary Baratta-Lorton, Addison Wesley

Workjobs II, Mary Baratta-Lorton, Addison Wesley

CDA mathematics materials:
Think, Talk, and Connect;
Think, Talk, and Read;
Common Sense in Arithmetic;
Developing Insights;
Drill and Practice at the Problem Solving Level;
Banking on Problem Solving;
Thursday Math, Robert W. Wirtz, Curriculum Development Associates

Problem Solving in Mathematics, Oscar Schaaf (Editor), Dale Seymour Publications

The Pattern Factory, Roper and Harvey, Creative Publications

TOPS Beginning Problem Solving, Dale Seymour Publications

Attribute Games and Problems, ESS, Webster/McGraw Hill

Real Math Thinking Story Books, Willoughby and others, Open Court

Make It Simpler, Meyer and Salee, Addison Wesley

National Council of Teachers of Mathematics (NCTM) Publications:
- Elementary Mathematics - What Parents Should Know About Problem Solving, Barbara Reys
- How to Choose and Create Good Problems for Primary Children, Nelson and Worth
- IDEAS from the Arithmetic Teacher - Grades 1-4

Washington State Mathematics Council (WSMC) Publications:
- Make It and Take It Math Games and Activities (1980 Single Topic Edition)
- Developing Thoughtful Behavior in Mathematics, John Firkins (1983 Single Topic Edition)

In addition to printed resources such as those given above, all of the manipulative materials listed for the other strands in this document should be available for use in problem-solving activities: e.g., collections of small objects, pattern blocks, etc.

I-B. GRADES 4-6

Learning Results	Instructional Implications
1. Students gather, categorize, and analyze data necessary for solving problems.	1.1 Teachers should encourage students to formulate clear questions as part of the problem-solving process.
	1.2 Teachers should provide situations in which the students must collect, organize, interpret, and evaluate data in order to generate as well as solve problems.
	1.3 Teachers should provide situations in which students must distinguish between relevant and irrelevant data.
2. Students select and apply appropriate strategies in problem situations.	2.1 Teachers should provide experiences from which students will learn to use problem-solving strategies such as: - looking for and using patterns; - constructing tables, charts, and graphs; - using estimation skills;

Learning Results	Instructional Implications
	- making organized lists; - acting out the problem situation; - hypothesizing, testing, and evaluating; - drawing a picture or making a model; - writing an equation; - solving a simpler or similar problem; - brainstorming; - looking for counter examples; - working backwards.
3. Students solve non-routine as well as routine problems.	3.1 Teachers should provide problem-posing as well as problem-solving opportunities from other disciplines: e.g., social studies, science, art, P.E., etc. 3.2 Teachers should provide an ample supply of non-routine problems throughout the mathematics program of the intermediate grades, so that skill in using the various problem-solving strategies undergoes continual development.

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Mindbenders, Midwest Publications

Critical Thinking, Book 1, Midwest Publications

Logic in Easy Steps, Midwest Publications

Syllogisms, A-C, Midwest Publications

Successful Problem-Solving Techniques, Carole Greene, Creative Publications

The Tokyo Puzzles, Martin Gardner, Creative Publications

After Math, Books I-IV, Seymour and others, Creative Publications

TOPS Developmental Workbooks, Imerzeel and others, Dale Seymour Publications

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IDEAS from the Arithmetic Teacher, Grades 1-4 and IDEAS from the Arithmetic Teacher, Grades 4-6, National Council of Teachers of Mathematics (NCTM)

Developing Thoughtful Behavior In Mathematics (1983 Single Topic Edition of Washington Mathematics), John Firkins, Washington State Mathematics Council (WSMC)

In addition to printed materials such as those listed above, a wide array of concrete manipulative materials (e.g., bean sticks, pattern blocks, etc.) are needed for successful implementation of a mathematics program that is developed from the guidelines contained in this document.

I-C. GRADES 7-8

Learning Results	Instructional Implications
1. Students understand and are able to use the various problem-solving strategies in dealing with applications of mathematics to "real life" situations that are relevant to their experiences and interests, as well as to non-routine problems, at an appropriate level of sophistication.	1.1 Teachers should provide learning experiences that emphasize problem-solving processes (as opposed to products) as well as the verbalization and communication of those processes. 1.2 Teachers should utilize small group dynamics to create a climate that will enhance active participation in the process of communicating and verbalizing problem-solving strategies and the discoveries that are made.

Learning Results	Instructional Implications
	1.3 Teachers should provide opportunities for developing the "read, explore, select a strategy, solve, review, and extend to similar cases" approach to "real life" applications of mathematics.
	1.4 Teachers should present problems for consideration that can be solved by more than one process and/or problems that have more than one solution.
	1.5 Teachers should make ample provision for the inclusion of non-routine problems throughout the grades 7-8 mathematics program such that skill in using most, if not all of the following strategies, is developed. - using patterns - constructing tables, charts, and graphs - making organized lists - modeling (e.g., acting it out) - hypothesizing, testing and evaluating - writing an equation - solving a simpler or similar problem - brainstorming - looking for counter examples - working backwards - AHA! (intuitive leaps)
	1.6 Teachers should provide experiences that emphasize the importance of estimation, approximation, and checking the reasonableness of answers in terms of the original problem situation.
	1.7 Teachers should encourage students to take advantage of the power of the calculator to enhance the range of problem situations that can be considered.

Learning Results	Instructional Implications
	1.8 Teachers should provide opportunities for the use of the computer as a problem-solving tool.
	1.9 Teachers should encourage the use of problem-solving processes in inter-disciplinary applications.
	1.10 Teachers should make use of logic problems as a means of encouraging logical thinking.

Bibliography of Selected Instructional Resources

Mathematics, A Human Endeavor, Harold R. Jacobs, W.H. Freeman and Company

Teaching Problem-Solving Strategies, Dolan and Williamson, Addison Wesley

Activities from the Mathematics Teacher, National Council of Teachers of Mathematics (NCTM)

IDEAS from the Arithmetic Teacher, Grades 6-8, National Council of Teachers of Mathematics (NCTM)

Multicultural Posters and Poster Activities (1982 Single Topic Edition of Washington Mathematics), National Council of Teachers of Mathematics (NCTM)

Developing Thoughtful Behavior in Mathematics (1983 Single Topic Edition of Washington Mathematics), John Firkins, Washington State Mathematics Council

When Are We Ever Gonna Have To Use This?, Hal Saunders, Dale Seymour Publications

Aftermath, Seymour and others, Creative Publications

TOPS, Greenes and others, Dale Seymour Publications

The Answer Is One, Robert E. Barns

Games, magazine, Playboy Publishing Co.

How To Develop Problem-Solving With A Calculator, Janet Morris, National Council of Teachers of Mathematics

Problem-Mathics, Greenes and others, Creative Publications

Wollywoggles and Other Creatures, Thomas O'Brien

Developing Skills in Estimation, Books A and B, Dale Seymour, Dale Seymour Publications

Problem Solving in Mathematics, Lane E.S.D. Project, Dale Seymour Publications

Access to calculators and computers is also needed as is a variety of manipulative materials.

II. K-8 MEASUREMENT STRAND

In Measurement In School Mathematics[1], Walter J. Sanders wrote, "The answer to 'Why measure?' is simple: measurement is necessary if we are to have the things we wish to have and to do the things we wish to do; the luxuries to which we are accustomed and do not wish to do without are a product of modern tehcnology, which is totally dependent on measurement; and many of the simple tasks that we all perform and take for granted are made possible or easier through measurement." Civilizations have been built upon advances in measurement understandings. In our own time, the navigational problems, alone, of a space flight require many precise measurements of distance, time, duration, relative position, size, weight, and speed. At a more personal level, decisions based upon measurement, whether consciously made or not, are a necessary part of our daily lives.

Measurement learning experiences for our students should be almost totally experiential. Hands-on measurement activities provide abundant opportunities for students to explore and learn from the world around them. A corollary to this is that measurement understandings and skills cannot be acquired from the textbook alone. Related considerations are:

- Young children need to develop an intuitive understanding of units of measure and of the approximate nature of measurement. Beginning understandings involve comparing, ordering, and the use of non-standard arbitrary units, often of the child's own choice (e.g., shoe length, weight of a jelly bean, etc.).
- Interdisciplinary applications of measurement should be emphasized.
- Attention should be given to the use of correct measurement notation and vocabulary so that this develops as a natural consequence of measurement activities.
- Approximately equal attention should be given to the use of customary units (inch, pound, etc.) and metric units. Except for rule-of-the-thumb approximations, conversions between systems should be avoided. (Such conversions as are necessary at the more advanced grade levels should be viewed as calculator activities.) While grades 7-8 students may be expected to know the distinction between "weight" and "mass", it is the view of the K-8 Writing Group that the term "weight" should be used with the distinction made only as it is necessitated by science applications at the upper grade levels. Note also that it was the expectation, some five to ten years ago, that the United States would now be well on the road to complete metrication. That this has not happened, nor is likely to happen in the near future, is now apparent; however, many texts produced in the '70s were totally metric and many of these are still in use. Teachers who use these texts must take this into account in the measurement activities they provide for their students.
- The relative precision of measurement tools should take into account the students' readiness to deal with the relative sizes of the units and their subdivisions. For example, centimeter rulers with millimeter markings or inch rulers with eighth-inch markings are not appropriate for use with primary children. The same considerations apply to durations of time, weight-size, etc.

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[1] "Why Measure?" (p.10), NCTM 1976 Yearbook

Learning Results	Instructional Implications
1. Students conserve length, volume, and weight (mass). (Piaget's principle of conservation)	1.1 Teachers should assess informally the students' ability to conserve length, volume, and weight to determine readiness for engaging in measurement activities.
	1.2 Teachers should provide activities that give students the opportunity to practice conservation skills.
2. Students compare and order distances (lengths), areas, volumes (capacities), weights, durations of time, and temperatures.	2.1 Teachers should provide a variety of concrete materials with examples and activities to establish understandings related to comparing and ordering distances (lengths), areas, volumes (capacities), weights, durations of time, and temperatures.
	2.2 Teachers should encourage the use of appropriate comparing and ordering terminology.
3. Students choose and use nonstandard units to determine measurements by estimating and counting.	3.1 Teachers should provide numerous opportunities for students as individuals or in small groups to estimate and measure distances (lengths), areas, volumes (capacities), and weights using a variety of nonstandard units.
4. Students choose and use appropriate tools of measurement to find distances (lengths), areas, volumes, weights, durations of time, and temperatures.	4.1 Teachers should provide activities leading to an understanding of the use of measurement tools such as centimeter and inch rulers, yardsticks and meter sticks, and trundle wheels.
	4.2 Teachers should provide activities involving standard measures of capacity, areas on an inch or centimeter grid (including areas on a geoboard), volumes of cube stacks, time duration using both digital and analog clocks and temperature.
5. Students recognize values of coins and use coins and bills in "real life" activities.	5.1 Teachers should provide a variety of "real life" activities involving the use of money at the appropriate levels of understanding.
6. Students apply measurement skills and understandings in problem-solving situations.	6.1 Teachers should involve students in problem-solving situations that require the use of measurement skills and understandings.

Bibliography of Selected Instructional Resources

Mathematics Their Way, Mary Baratta-Lorton, Addison Wesley

Measure Matters (A, B), Carolyn Aho and others, Creative Publications

Shape and Size, Nuffield Mathematics Project, John Wiley and Sons

Measurement in School Mathematics, NCTM 1976 Yearbook, National Council of Teachers of Mathematics

IDEAS from The Arithmetic Teacher, Grades 1-4, National Council of Teachers of Mathematics

Developing Thoughtful Behavior in Mathematics, 1983 Single Topic Edition of Washington Mathematics, John Firkins, Washington State Mathematics Council

Additional Resources

Objects for comparison weighing

Metric and traditional weights

Balance board

Balance scale (pan balance)

Spring scale

Customary and metric rulers, "sticks" and tapes

Customary and metric trundle wheels

String or yarn

Variety of materials for use as non-standard units (paper clips, marshmallows, cubes, etc.)

Blocks or cubes of various sizes

Pattern blocks

Maps

Non-standard, traditional, and metric containers for measuring capacity

Cookbooks

Analog and digital clocks

Play clocks

Stop watch

Calendar

Number lines

Money

Cuisenaire Rods

Multi-links

Unifix cubes

Geoboards

Inch and centimeter grid paper

Playdough or clay

II-B. GRADES 4-6

Learning Results	Instructional Implications
1. Students select and use the appropriate traditional or metric devices to determine length (distance), area, volume (capacity), weight, time, and temperature.	1.1 Teachers should provide opportunities for students to choose and use the appropriate customary and metric devices in measurement activities.
	1.2 Teachers should provide instruction in reading a 12-hour clock face and in writing digital time, using a.m. and p.m.
	1.3 Teachers should provide opportunities for students to compute elapsed time in terms of hours, days, years, centuries, etc.
2. Students convert measurements in given units to equivalent measurements in other units within a system (not between systems).	2.1 Teachers should provide instruction in converting measurements within a system (ounces to pounds, centimeters to meters, etc.), and provide activities which require that such conversions be made.
3. Students estimate measurements in non-standard, traditional and metric units, checking the estimate by measuring with the appropriate measurement tool.	3.1 Teachers should provide activities that require students to estimate measurements (e.g., visualizing the length of an object in terms of handspans, paper clips, inches, meters, etc.)
	3.2 Teachers should provide activities that lead to understandings related to the approximate nature of measurements.

Bibliography of Selected Instructional Resources

Mathematics - A Way of Thinking, Robert Baratta-Lorton, Addison Wesley

This Book Is About Time, Marilyn Burns, Creative Publications

Unified Science and Mathematics in the Elementary School, National Science Foundation

Shape and Size, Nuffield Mathematics Project, John Wiley and Sons

Design for Math, Northwest Math Designers

Measurement in School Mathematics, NCTM 1976 Yearbook, National Council of Teachers of Mathematics

Developing Thoughtful Behavior in Mathematics, WSMC 1983 Single Topic Edition of Washington Mathematics, Washington State Mathematics Council

Additional Resources

- Traditional and metric rulers, "sticks" and tapes
- Traditional and metric trundle wheels
- Traditional and metric weights
- Collection of objects to be weighed
- Traditional and metric capacity measuring devices including graduated beakers and cylinders
- Objects for use as non-standard units of length, weight, etc.
- Scales of various types
- Thermometers
- Calendar
- Boxes, cylinders, and other solids for hands-on measurement activities
- Grid paper (inch and centimeter)
- Clocks: analog and digital
- Maps (park, city, state, etc.)
- Cookbooks
- Directional Compass
- Geo-strips
- Geoboards
- Tangrams
- Polygons (plastic or cardboard)
- Pattern Blocks
- Number lines
- Clinometer
- Sets of colored cubes

II-C. GRADES 7-8

Learning Results	Instructional Implications
1. Students understand measurement concepts and are skilled in using both traditional and metric units of length, area, capacity/volume, mass/weight, time, and temperature.	1.1 Teachers should use correct symbolism and vocabulary, instruct students in correct usage, and provide activities that emphasize the need for correct usage. 1.2 Teachers should provide opportunities for the students to select the appropriate units of measurement as well as the most useful measuring instrument. 1.3 Teachers should provide ample opportunity for students to estimate lengths, areas, capacities/volumes, mass/weight, time, and temperature and for students to check the reasonableness of measurement results.

Learning Results	Instructional Implications
	1.4 Teachers should provide experiences with manipulatives that foster understanding of the concepts of length, area, and volume.
	1.5 Teachers should introduce measurement activities that are relevant to the personal experiences of students.
	1.6 Teachers should present activities that require the use of problem-solving skills and measurement skills in collecting, organizing, and analyzing data.
	1.7 Teachers should encourage the coordination of measurement instruction and activities across the disciplines.

Bibliography of Selected Instructional Resources

Developing Skills in Estimation, Books A and B, Dale Seymour, Dale Seymour Publications

Measurement in School Mathematics, NCTM 1976 Yearbook, National Council of Teachers of Mathematics

Additional Resources

Customary and metric measurement tools: rulers, beakers, balance and spring scales, thermometers, etc.

Metric cookbooks

Maps

Grid paper: customary and metric

Geoboards

Canadian newspapers

Collection of boxes, cylinders, and other objects for measurement activities

III K-8. GEOMETRY STRAND

The development of informal geometric concepts and understandings is a most important part of the K-8 mathematics curriculum. Initial geometric ideas and perceptions evolve from the child's observation of his/her world. Classroom instruction should build upon these observations. It is an area of mathematics that encourages exploration and discovery. Relationships between geometry and art should be exploited to provide pleasurable opportunities for students to use their imagination and creativity. The perceptive teacher will also recognize connections between geometry and other subject areas such as P.E., science, home and family life, social studies and industrial arts which can lead to activities of relevance and interest to students. Recognition should be given to geometric form in nature as well as in human constructions. Children should be afforded many opportunities to enhance and extend their spatial skills through manipulating and arranging objects such as colored cubes and pattern blocks, building castles and towers and whatever else their imagination dictates. Older children may analyze the geometric patterns in the art of the Native American or of other cultures and create patterns based on this analysis. Grades 7-8 students may wrestle with topological ideas that will intrigue them. There is truly no end of topics for investigation by individual students or students working together in small groups within this area of mathematics.

III-A. GRADES K-3

Learning Results	Instructional Implications
1. Students recognize the attributes of two- and three-dimensional figures.	1.1 Teachers should provide activities in which students see, feel, and describe geometric forms.
	1.2 Teachers should provide activities that encourage students to identify the attributes of geometric forms and to classify them according to these attributes.
	1.3 Teachers should utilize physical movement activities to enhance spatial visualization: e.g., "Imagine you are inside a giant beach ball. Move your hands all around to touch the inside of the ball."
	1.4 Teachers should provide opportunities for students to build structures with blocks, pattern blocks and other manipulatives to enhance spatial visualization.

Learning Results	Instructional Implications
	1.5 Teachers should provide opportunities for students to explore the attributes of two- and three-dimensional objects through the use of templates, geo-strips, logo computer programs, etc.
2. Students develop spatial visualization concepts associated with symmetry, similarity, and congruence.	2.1 Teachers should provide activities utilizing geoboards and other manipulatives, paper folding and cutting activities, etc. that lead to an understanding of the concepts of symmetry, similarity, and congruence.
	2.2 Teachers should lead students to see that the relative position or orientation of objects does not affect the congruence of geometric figures.
	2.3 The teacher provides opportunities for students to discover tessella-tions of congruent shapes through the use of manipulatives such as pattern blocks.
	2.4 Teachers should encourage students to find examples of symmetry, similarity, congruence, and the tessellation of congruent shapes in their environment.
3. Students understand and use grid systems.	3.1 The teacher provides opportunities for students to gain an intuitive understanding of grid systems through the use of games such as Battleship and Tic Tac Toe, making geoboard designs, reading maps, and making graphs.

Bibliography of Selected Instructional Resources

Mathematics Their Way, Mary Baratta-Lorton, Addison Wesley

IDEAS from the Arithmetic Teacher, National Council of Teachers of Mathematics

Shape and Size, Nuffield Mathematics Project, John Wiley and Sons

Developing Thoughtful Behavior in Mathematics, WSMC 1983 Single Topic Edition, John Firkins, Washington State Mathematics Council

Other Resources

Manipulatives	Activities and Games
Titles	Paperfolding (Origami)
Pattern blocks with mirrors	Logo computer programs
Attribute blocks	Maps
Geoblocks	Pattern block games
Geoboards/dot paper	Soma Cube and other spatial concept puzzles
Geostrips	
Boxes, cylinders and other geometric solids	Battleship, Tic Tac Toe, and other grid games
Colored cubes	Pentominoes
Tangrams	

III-B. GRADES 4-6

Learning Results	Instructional Implications
1. Students recognize, label, compare, and describe the attributes of geometric figures.	1.1 Teachers should present activities in which students handle and describe a variety of geometric shapes, identify two- and three-dimensional shapes from verbal descriptions, and construct and identify symmetrical and asymmetrical structures or designs.
	1.2 Teachers should present activities in which students perform simple geometric constructions using compass and straightedge.
	1.3 The teacher presents activities in which the students exhibit understanding of such concepts as similarity and congruence, parallelism and perpendicularity, etc.
2. Students understand the concepts of perimeter, area, and volume and apply these understandings in problem situations.	2.1 Teachers should provide activities that lead to an understanding of perimeter, area, and volume (e.g., direct measurement of perimeters, counting unit squares and unit cubes).

Learning Results	Instructional Implications
	2.2 Teachers should provide activities in which students measure the dimensions of simple two- and three-dimensional figures and use the results to compute areas and volumes.
3. Students demonstrate an understanding of spatial relationships.	3.1 Teachers should provide activities in which students; Build solid shapes with blocks and draw front, top, and side views. Build three-dimensional shapes from two-dimensional representations. Build three-dimensional shapes from written directions.
	3.2 Teachers should provide activities in which students are required to make translations of figures in the plane: e.g., flips, slides, rotations, enlargements and reductions of figures on a grid.
	3.3 Teachers should provide activities in which students are led to recognize and extend geometric patterns, and to create geometric patterns of their own design including tessellations.
	3.4 Teachers should provide opportunities for students to interpret and make scale drawings/maps of real objects/actual places and make use of interdisciplinary applications of these skills (social studies, shop classes, etc.)
	3.5 Teachers should make use of optical illusions to demonstrate that things are "not always what they seem."

Bibliography of Selected Instructional Materials

Ideas from The Arithmetic Teacher, Grades 1-4 and Grades 4-6, National Council of Teachers of Mathematics

Developing Thoughtful Behavior in Mathematics, WSMC, 1983, Single Topic Edition, John Firkins, Washington State Mathematics Council

Dover Pictorial Archive Series (copyright free), Dover Publications, Catalog 99062-3 (free), Includes books such as the following:

American Indian Design and Decoration, Leroy Appleton

Japanese Optical and Geometric Art, Hajime Ouchi

Snow Crystals, Bently and Humphreys

- and much more.

Tangramath, Dale Seymour, Creative Publications

Seeing Shapes, Ernest R. Ranucci, Creative Publications

Creating Escher Type Drawings, Ernest R. Ranucci, Creative Publications

Shape and Size, Nuffield Mathematics Project, John Wiley and Sons

Tessellations: The Geometry of Patterns, Bezuszka and others, Creative Publications

Geometry and Visualization, Mathematics Resource Project, Creative Publications

Attribute Games and Problems, ESS, Webster Division/McGraw Hill

Math for Smarty Pants, Marilyn Burns, Yolla Bolly Press

Make It and Take It Math Games and Activities, WSMC 1980 Single Topic Edition, Washington State Mathematics Council

Multicultural Mathematics Posters and Poster Activities, WSMC 1982 Single Topic Edition, National Council of Teachers of Mathematics

Multicultural Mathematics Materials, Marina C. Krause, National Council of Teachers of Mathematics

Other Resources

Manipulatives:

Geoboards and dot paper

String, yarn, colored rubber bands

Tracing paper

Straws and pipe cleaners

Tangrams

Various size cubes (wooden, plastic, foam)

Models of geometric shapes (two- and three-dimensional)

Attribute blocks

Pattern blocks/parquetry blocks

Pentominoes/Soma cubes

Boxes and large cutout two-dimensional shapes (cardboard or wood) for volume and area activities

Games and Activities:

Pattern block games

Pent-Up (See Make It and Take It Math Games and Activities)

Great Shapes Contest (See Make It and Take It Math Games and Activities)

Dissection puzzles

String art

Oragami/paper folding and cutting

Learning Results	Instructional Implications
1. Students recognize geometric figures and understand and apply geometric concepts.	1.1 Teachers should use the vocabulary and symbolism that is appropriate to geometry, instruct students in correct usage, and expect students to understand and use the vocabulary and symbolism correctly.
	1.2 Teachers should provide instructional activities, using concrete models, that emphasize the recognition and differentiation of the common geometric plane and solid figures with exposure to less common figures.
	1.3 Teachers should provide an abundance of experiential activities involving manipulatives such as geoboards, tangrams and other dissection puzzles, mirrors, pattern/parquetry blocks; and activities involving tessellations, Escher tilings, string art, flexagons, paper folding and cutting, model building, etc.
	1.4 Teachers should provide experiences in using protractors, rulers and compasses for measuring and constructing geometric figures.
	1.5 Teachers should provide opportunities for students to perform common geometric constructions with compass and straightedge only, and to create designs based on these constructions.
	1.6 Teachers should provide activities that relate geometry to the students' environment (both natural and human-made) and to the requirements of everyday life.
	1.7 Teachers should utilize computer graphics such as LOGO and other computer programs as appropriate to develop geometric understandings.
	1.8 Teachers should provide instruction, coordination, and activities that relate geometry to the other disciplines (shop, art, etc.)

Learning Results	Instructional Implications
	1.9 Teachers should use every opportunity to present activities that encourage students to explore geometric ideas, and make discoveries such as the relationship among faces, edges, and vertices of polyhedra, the rule for traversibility of networks, etc.
2. Students understand the concepts of perimeter (circumference), area, and volume and apply such understanding in solving problems involving perimeter, area, and volume.	2.1 Teachers should provide exploratory experiences at the concrete level that lead to an understanding of the concepts of perimeter, area, and volume, enabling students to find the measures of each prior to the introduction of formulas.
	2.2 Teachers should provide exploratory experiences from which students can discover the relationship between the circumference of a circle and its diameter, and between the area of a circle and its radius.
	2.3 Teachers should utilize the results of the activities in 2.1 and 2.2 above to help students generalize the associated formulas.
	2.4 Teachers should present problems of relevance to students in which the students are required to use the appropriate formulas for perimeter, circumference, area, and volume; and extending the range of "real life" problems that can be considered by encouraging the use of calculators to minimize computational "hang-ups."
	2.5 Teachers should provide activities that require the use of problem-solving strategies to develop such ideas as Pick's Theorem, the Pythagorean Theorem, etc.

Bibliography of Selected Instructional Resources

Geometry and Visualization, Mathematics Resource Project, Creative Publications

Tessellations: The Geometry of Patterns, Bezuszka and others, Creative Publications

Creating Escher Type Drawings, Ranucci and Teeters, Creative Publications

Tangramath, Dale Seymour, Creative Publications

Geoboard activities from a variety of publishers

Developing Thoughtful Behavior in Mathematics, WSMC, 1983, Single Topic Edition, John Firkins, Washington State Mathematics Council

Make It and Take It Math Games and Activities, WSMC, 1980, Single Topic Edition, Washington State Mathematics Council

Multicultural Mathematics Posters and Poster Activities, WSMC, 1982, Single Topic Edition, National Council of Teachers of Mathematics

Multicultural Mathematics Materials, Marina C. Krause, National Council of Teachers of Mathematics

Seeing Shapes, Ernest R. Ranucci, Creative Publications

The Mathematics of Islamic Art, Metropolitan Museum of Art, National Council of Teachers of Mathematics

Activities from The Mathematics Teacher, National Council of Teachers of Mathematics

IDEAS from The Arithmetic Teacher, National Council of Teachers of Mathematics

Dover Pictorial Archive Series, (copyright free), Dover Publications, Catalog 59062-3 (free). Includes books such as the following:

American Indian Design and Decoration, Leroy Appleton

Japanese Optical and Geometric Art, Hajime Ouchi

Islamic Geometrical Pattern and Design, J. Bourgoin

Snow Crystals, Bently and Humphreys

- and many more.

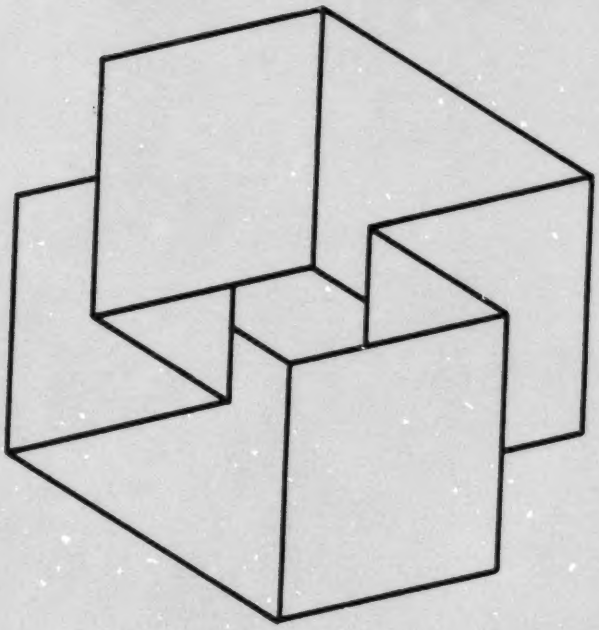
Other Resources

Manipulatives:

- Geoboards and dot paper
- Pentominoes, Soma cubes
- Tangrams
- Compasses
- Protractors
- Rulers/straight edges
- Grid paper
- Geometric solids
- Mirrors
- Pattern blocks/parquetry blocks
- Attribute blocks
- 2-dimensional "cutouts" (wood, cardboard, plastic)

Games and Activities:

- Tessellations
- Escher tilings
- String art
- Flexagons
- Polyhedra construction
- Pent-Up (pentomino game)
- Geometric art from other cultures
- Oragami
- Dissection puzzles (tangrams, etc.)



IV. K-8 NUMBER PROPERTIES, THEORY, AND COMPUTATION SKILLS

The attainment of reasonable proficiency in pencil/paper computation skills remains an important goal in K-8 mathematics; however, it must be recognized that the calculator has reduced dependence on such methods.* At the same time, the use of the calculator for computational purposes demands increased attention to the development of skills in estimation and in the assessment of reasonableness of answers. This in turn means that we must put greater emphasis on the development of facility with the basic number facts and mental arithmetic. An understanding of and practice with the computational algorithms should be accomplished within the settings of mathematical investigations, problem solving, and game-type activities of interest to students. Repetitious and isolated drill and practice is relatively unproductive in terms of understanding and skill retention; and it usurps time that could be better spent on problem solving and mathematical investigations in the other strands on which our instructional energies should be focused. Additional important considerations are:

- The most important task at the primary level is to lead children to a conceptualization of number. Teachers need to provide many exploratory experiences at the concrete level. During this stage of "concrete" activity, it is important for children to talk about what they are doing, what they have discovered, and what the relationships are that they begin to observe. As they describe their experiences in words that make sense to them, they engage in a process of internalizing the concepts, a necessary step in their progression toward higher-level understandings.

- Too little time in many classrooms currently is spent developing place-value understandings. Too often, the practice is to move quickly to abstract representation of multi-digit numerals with whatever conceptual help is afforded by textbook illustrations. This practice does <u>not</u> allow students time to internalize the concepts involved - a <u>must</u> before work with the computation algorithms is attempted.

- Students at <u>all</u> grade levels best learn new concepts through a progression of activities that proceed from the concrete to the pictorial level, and only then to the abstract level. Since instruction that makes use of concrete and visual materials is an unfamiliar approach for many teachers, particularly beyond the primary grades, there is need for inservice to provide all teachers of mathematics with the necessary instructional "tools."

- While uses of the calculator as an aid in computation are obvious, its uses in concept development are less widely understood. This is also an area in which teacher education is required.

- The area of number theory offers a multitude of opportunities for investigation, discovery, and problem solving. It is deserving of increased attention, particularly at the middle school level.

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*There are ever-increasing numbers of mathematics educators who advise using calculators for nearly all of the computation tasks that have long been a mainstay of the K-8 mathematics curriculum. Among other things, they point with horror at the fact that the approximate equivalent of one year of mathematics instruction is devoted to the division algorithm.

Learning Results	Instructional Implications
1. Students demonstrate conservation of number. (Piaget's principle of conservation)	1.1 Teachers should assess each student's ability to conserve number through a variety of conservation assessment activities, recognizing that until he/she has reached this maturation level, further number work will be unproductive.
2. Students identify and order numerals; and recognize number patterns and relationships between/among numbers.	2.1 Teachers should provide frequent opportunities for students to count concrete objects in sets and to identify the numeral for the number of objects in each set enabling the student to conceptualize the number to which the numeral relates.
	2.2 Teachers should provide many opportunities for students to create and compare equal and unequal sets and to incorporate the language of "greater than," "less than," and "equal to" into their vocabulary.
	2.3 Because of the importance of patterns in mathematics, teachers should provide numerous opportunities for students to discover numerical patterns and relationships through the use of concrete materials and to talk about their discoveries.
	2.4 As teachers move children from the concrete into the symbolic representation of number patterns and relationships, it is important to continue to use the language developed in the concrete stage.
3. Students understand the properties and operations of whole numbers and apply them in generating and solving problems.	3.1 Teachers should have a thorough knowledge of cognitive development in order to recognize students' readiness for experiences with operations and properties of operations.
	3.2 Teachers should provide numerous investigative experiences at a concrete level before symbols are used or paper/pencil work is attempted; experiences should include manipulation, verbalization, and demonstration in the cases of adding on, removing (covering or hiding), making multiple sets, and separating into subsets of equal number (division).

Learning Results	Instructional Implications
	3.3 Teachers should provide many concrete experiences with number operations so that the student will develop an intuitive understanding of the commutative, associative, and identity properties of addition and multiplication.
	3.4 Teachers should expect students to memorize addition and subtraction basic facts only when they demonstrate conceptual understanding of the associated operations; however, the need for "instant recall" deserves continuing attention throughout the grades.
	3.5 Teachers should provide abundant opportunities for students to develop informal mental arithmetic skills in which number patterns rather than formal algorithms are used to determine approximate and/or exact answers.
	3.6 Teachers should provide much regrouping experience at the concrete level before the standard algorithms for addition and subtraction are introduced.
	3.7 Teachers should have students use calculators to check estimates, expedite time-consuming computation, and develop number concepts and patterning.
	3.8 Teachers should regularly provide students with problem-solving opportunities which draw upon the students' knowledge of number concepts, number relationships, and computation.
4. Students understand the concept of fractional parts of a whole object.	4.1 Teachers should provide numerous opportunities for students to divide objects into fractional parts.
	4.2 Teachers should use real objects and pictorial representations to lead students to an understanding of the meaning of simple fraction numerals.

Learning Results	Instructional Implications
5. Students understand place value concepts and can use the standard algorithms that are dependent upon this understanding.	5.1 Teachers should help students develop place value understandings throughout the primary grades by providing many activities over an extended period of time in which students group objects into sets of 10 (or some other arbitrary grouping number), record the number of 10's and leftover 1's, and relate the result to the standard 2-digit numeral with the extension of this type of activity to regrouping 10's as 100's when students are ready.
	5.2 Teachers should base instruction in the standard algorithms on the use of manipulative materials such as bean sticks, base ten blocks or similar concept-building manipulatives, enabling the students to move from the concrete through the representational to the abstract as they are ready to do so.
6. Students estimate to determine if the solution is reasonable.	6.1 Teachers should help students develop confidence in estimating by providing them with many opportunities to "guesstimate" and check (an excellent place to use calculators).
	6.2 Once students are competent in and comfortable with estimating, teachers should expect them to use estimates to assess the reasonableness of computation results.
7. Students solve problems that require the use of computation skills.	7.1 Teachers should provide problem situations in which students are required to choose the operation and/or devise a strategy before the computation can be completed to find solutions.

Bibliography of Selected Instructional Resources

Mathematics Their Way, Mary Baratta Lorton, Addison Wesley

Workjobs II, Mary Baratta-Lorton, Addison Wesley

Developing Computational Skills NCTM 1978 Yearbook, National Council of Teachers of Math.

IDEAS from The Arithmetic Teacher, National Council of Teachers of Mathematics

Classroom Ideas on Research on Computational Skills, Suydam and Dessart, National Council of Teachers of Math.

Make It and Take It Math Games and Activities, WSMC 1980 Single Topic Edition, Washington State Mathematics Council

Real Math Thinking Story Books, Willoughby and others, Open Court

Computation and Structure (2-3) Nuffield Mathematics Project, John Wiley and Sons

CDA mathematics materials:
Think, Talk, and Read,
Think, Talk, and Connect,
Common Sense in Arithmetic,
Insights into Mathematics,
Drill and Practice at the Problem Solving Level,
Banking on Problem Solving,
Robert W. Wirtz, Curriculum Development Associates

Other Resources

Manipulatives:

- Wood cubes
- Unifix cubes
- Pattern blocks
- Math balance
- Beans, portion cups, bean sticks
- Collections of small items for counting such as toothpicks, plastic counters, tiles, beads, keys, etc.
- Digit cards, number wheel response cards
- Chips, different colors
- Hundred charts, blank 10-by-10 grids
- Base ten blocks
- Number lines

Manipulatives, continued:

- Dice (number cubes)
- Cuisenaire Rods
- Graph paper
- Fraction bars
- Circles, squares, etc. to be cut into fractional parts
- Calculators

Games and Activities:

- Dominoes
- Number line games
- Contig, Multrix, Sumtrix, and other computation games
- Chip Trading and other place value games
- Real Math Game Mats, Open Court

Learning Results	Instructional Implications
1. Students understand and use the properties of numbers and properties of operations.	1.1 Teachers should provide experiences in which students must identify equality and inequality of numbers, compare numbers, and order numbers in cases that include more than +1 or -1 patterning.
	1.2 Teachers should provide experiences at the concrete level to insure that students understand the concepts of addition, subtraction, multiplication, and division before any attempt is made to introduce the algorithms that are used to find sums, differences, products and quotients.
	1.3 Teachers should provide experiences that lead students to discover and use the commutative, associative, inverse, and identity properties.
2. Students memorize the basic number facts.	2.1 Teachers should instruct the students in the use of a variety of strategies for memorizing the basic number facts and provide sufficient practice to assure retention so that they will be able to estimate and compute successfully.
3. Students understand and use algorithms for addition, subtraction, multiplication, and division involving whole numbers and decimals.	3.1 Teachers should introduce and/or reteach place value concepts through the use of concrete manipulatives (which may include work in bases other than ten), proceeding to the pictorial stage and on to the abstract stage only when students demonstrate understanding. This progression will enable students to: a. identify the value of each digit in a numeral; b. represent a numeral in expanded form; c. use a decimal point in writing numerals; d. read and write numerals from millions to ten-thousandths; e. acquire an understanding of the addition, subtraction, multiplication, and division algorithms.

Learning Results	Instructional Implications
	3.2 Teachers should introduce and/or re-teach the computational algorithms through the use of concrete manipulatives within the context of problem situations, requiring students to describe and record both the processes used and the results obtained in their efforts to solve the problems.
	3.3 Only when students demonstrate complete understanding of the algorithms through successful completion of activities as indicated in 3.2 above should the teacher provide problem-solving activities involving computation without the use of manipulative or pictorial aids (i.e., at the abstract level).
	3.4 Teachers should also make use of calculators in the development of understandings related to place value and the computation processes.
4. Students understand fractional concepts at the concrete and pictorial levels including comparing and ordering, and can find sums, differences and products of fractions.	4.1 Teachers should provide a variety of manipulative experiences that lead students to an understanding of fractions as parts of wholes, and extend these experiences to include equivalence, inequality (comparing and ordering), and methods of finding sums, differences, and products of fractions. No attempt should be made to teach the abstract algorithms until the students demonstrate understanding at the concrete and pictorial levels.
5. Students use estimation to predict results of computation and to assess the reasonableness of answers.	5.1 Teachers should provide numerous opportunities for students to solve problems involving whole numbers, decimals, and fractions when the only "answers" that are required are estimated ones.
	5.2 Teachers should stress the need to assess answers in terms of "reasonableness" through application of the skills of rounding and estimating, and provide opportunities for the students to exercise these skills.

Learning Results	Instructional Implications
	5.3 Teachers should provide activities in which students round the results of computation to obtain approximations with the degree of precision that is appropriate to the problem situation.
6. Students use calculators and/or computers in solving problems.	6.1 Teachers should provide instruction in the use of calculators and require their use in repetitious computations and/or those that are otherwise complex and time consuming.
	6.2 Teachers should provide an abundance of "real life" situations in which the students use calculators to complete all of the necessary computations.
	6.3 Teachers should provide various problem-solving situations in which calculators/computers can be used to advantage in the problem-solving process.
7. Students understand some basic number theory ideas and solve problems involving these ideas.	7.1 Teachers should provide activities that develop some of the simpler number theory ideas (e.g., odds/evens, factors/multiples, primes/composites).
	7.2 Teachers should involve students in problem-solving activities in which number theory understandings are used.

Bibliography of Selected Instructional Resources

Mathematics, A Way of Thinking, Robert Baratta-Lorton, Addison Wesley

Developing Insights into Elementary Mathematics: Operations with Whole Numbers, Robert W. Wirtz, Curriculum Development Associates

Math for Smarty Pants, Marilyn Burns, Yolla Bolly Press

Picturing Arithmetic - From Models to Symbols, Madell and Stahl, Creative Publications

Number Sense and Arithmetic Skills, Mathematics Resource Project, Creative Publications

Egg-citing Fractions, Edward C. Beardslee, Enrich, Inc.

Fractions with Pattern Blocks, Mathew E. Zullie, Creative Pub.

Beginning Calculator Math, Vervoort and Mason, Fearon Pitman Pub.

Calculator Math, Edward C. Beardslee, Enrich, Inc.

Classroom Ideas from Research on Computational Skills, Suydam and Dessart, National Council of Teachers of Mathematics

IDEAS from The Arithmetic Teacher, National Council of Teachers of Mathematics

How to Develop Problem Solving Using a Calculator, Janet Morris, National Council of Teachers of Mathematics

Elementary School Mathematics: What Parents Should Know about Estimation, Barbara Reys, National Council of Teachers of Mathematics

A New Twist, Developing Arithmetic Skills Through Problem Solving, Pederson and Armbruster, Addison Wesley

Make It and Take It Math Games and Activities, WSMC, 1980, Single Topic Edition, Washington State Mathematics Council

Fraction Bars, Davidson and Bennett, Creative Publications

Decimal Squares, Albert B. Bennett, Creative Publications

Shape and Size, Nuffield Mathematics Project, John Wiley and Sons

Other Resources

Manipulatives:

- Beads, beans, cups, bowls, etc.
- Ceramic tiles
- Number lines
- Digit cards/cards 1-20
- Multiplication grid sheets
- Bean sticks
- 0-99 matrix
- Blocks and cubes
- Abacus
- Craft sticks/sorting kits
- Multi-fact cards
- Lines and crossing points flash cards
- Dice/number cubes
- Cuisenaire Rods
- Decimal squares
- Egg cartons
- Pattern blocks
- Praction blocks
- Fraction "pies" and bars
- Geoboards
- Chips, and chip trading boards
- Play money
- Place value blocks
- Metric measuring tapes
- Counting frames
- Graph paper
- Decimal strips
- Calculators

Games and Activities:

- Bottle Cap Math
- Egg Carton Math
- Fraction Card Games
- Fraction Dominoes
- Paper Folding
- Chip Trading
- Real Math Game Mats, Open Court
- Games and Activities from Make It And Take It Math Game and Activities, WSMC 1980 Single Topic Edition

Learning Results	Instructional Implications
1. In operations and processes involving whole numbers, students develop self-sufficiency in utilizing properties of operations and the rules of order of operations, in rounding and estimating, and in using calculators.	1.1 Teachers should emphasize properties of operations and the rules of order of operations in the review of whole number computation skills with the aid of calculators.
	1.2 Teachers should provide practice in rounding numbers to aid students in estimating and in determining the reasonableness of answers as is necessary when calculators are used to complete computations.
	1.3 Teachers should utilize games and other motivational activities to stress instant recall of the basic number facts, speed and accuracy in mental computation, and facility with appropriate pencil/paper computation skills. Among the activities that may be used are those involving sequences, patterns, palindromic numbers, and magic squares.
	1.4 Teachers should provide students with activities that lead to the understanding and use of exponential notation.
	1.5 Teachers should use the vocabulary and symbolism of mathematics, instruct students in correct usage, and expect them to use the vocabulary and symbolism correctly.
2. Students understand the structure of the decimal system of numeration and use decimals in computation, "real world" applications and problem solving.	2.1 Teachers should provide experiences ranging from the concrete through the pictorial to the abstract which illustrate the meaning of place value in the decimal system of numeration.
	2.2 Teachers should provide relevant and concrete experiences that lead to generalization of the rules for placement of the decimal point in computation exercises.
	2.3 Teachers should provide activities that require students to order and compare decimals whether or not they have the same number of decimal places.

Learning Results	Instruction Implications
	2.4 Teachers should provide practice in the rounding of decimals to aid the student in estimating and determining the reasonableness of answers.
	2.5 Teachers should provide experiences with decimals as they apply to our money system (including unit pricing and sales tax) and to metric measurement.
	2.6 Teachers should provide students with activities that lead to an understanding and use of exponential notation.
	2.7 Teachers should encourage the use of calculators in real-world applications and problem-solving activities when the focus of the activity is on the application or problem-solving process rather than the computational skill involved.
3. Students understand fraction concepts, the relation of fractions to decimals, computation algorithms involving fractions, and can use fractions in "real world" applications and problem solving.	3.1 Teachers should provide experiences ranging from the concrete through the pictorial to the abstract that illustrate the meaning of fractions, and the relationship of decimals to fractions.
	3.2 Teachers should present activities that lead students to understand how to compare and order fractions and how to change them to equivalent forms.
	3.3 Teachers should provide experiences ranging from the concrete through the pictorial to the abstract that develop an understanding of the algorithms, restricting the complexity of the fractions involved in such computations to those that are relevant to student experience (e.g., halves, thirds, fourths, fifths, eighths, tenths, and twelfths).
	3.4 Teachers should provide real-life applications and problem-solving experiences that involve fractions.

Learning Results	Instructional Implications
4. Students understand the meaning of ratios and proportions and can apply them in solving problems.	4.1 Teachers should provide comparison experiences from the concrete through the pictorial to the abstract that lead to an understanding of ratios and equivalent ratios.
	4.2 Teachers should provide experiences that lead to an understanding of proportions and how to solve them.
	4.3 Teachers should provide a variety of "real life" problem situations that require the use of proportions in their solutions.
5. Students understand the meaning of percents, their relation to fractions/decimals, and can apply these understandings in "real life" problem situations.	5.1 Teachers should provide activities involving circle graphs and other pictorial aids to enhance the understanding of percent.
	5.2 Teachers should provide concrete experiences illustrating the relationship among fractions, decimals and percents and between ratios and percents.
	5.3 Teachers should emphasize the uses of percents that have everyday relevance including percents of increase/decrease, percent of discount, simple interest, percent of games won/lost, etc.
	5.4 Particularly in the use of nonintegral percents, teachers should encourage the use of calculators in solving percent problems.
	5.5 Teachers should encourage the coordination of instruction in the use of percents with other disciplines such as social studies.
	5.6 Teachers will provide instruction in the use of various methods for solving percent problems including the use of proportions, conversion to fractions/decimals and the use of formulas.
6 Students investigate number theory ideas, make conjectures as a result of their investigations, and see whether or not they can "prove" their conjectures.	6.1 Teachers should provide many opportunities for students to investigate and use ideas involving factors/multiples, greatest common factors/least common multiples, primes/composites, and rules of divisibility.

Bibliography of Selected Instructional Resources

How to Develop Problem Solving with a Calculator, Janet Morris National Council of Teachers of Mathematics

Activities from The Mathematics Teacher, National Council of Teachers of Mathematics

IDEAS from The Arithmetic Teacher, Grades 6-8, National Council of Teachers of Mathematics

Developing Computational Skills, NCTM 1978 Yearbook, National Council of Teachers of Mathematics

Mathematics for the Middle Grades (5-9), NCTM 1982 Yearbook, National Council of Teachers of Mathematics

Fraction Bars by Bennett and Davidson, Creative Publications

Fractions with Pattern Blocks, Mathew E. Zullie, Creative Pub.

Decimal Squares, Alfred B. Bennett, Creative Publications

Classroom Ideas from Research on Computation Skills, Suydam and Dessart, National Council of Teachers of Mathematics

Make It and Take It Math Games and Activities, WSMC 1980 Single Topic Edition, Washington State Mathematics Council

Multicultural Mathematics Posters and Poster Activities, WSMC 1982 Single Topic Edition, National Council of Teachers of Mathematics

Number Sense and Arithmetic Skills, Mathematics Resource Project, Creative Publications

Aftermath, Dale Seymour, Creative Publications

Estimation, Books A and B, Dale Seymour, Dale Seymour Publications

Egg-citing Mathematics, Edward C. Beardslee, Enrich, Inc.

A New Twist, Developing Arithmetic Skills Through Problem Solving, Pedersen and Armbruster, Addison Wesley

Other Resources

Manipulatives:

- Fraction bars, pattern blocks
- Graph paper
- Egg cartons
- Geoboards
- Base ten blocks
- Decimal strips
- Play money
- Sales catalogs/newspapers

Games and Activities:

- Games, commercial or teacher developed (see Make It and Take It Math Games and Activities)
- Paper folding
- Basic facts/fraction card games
- Prime Drag (Creative Pub.)
- Factor games

Other:

- Calculators and computers

V. K-8 PROBABILITY AND STATISTICS STRAND

The National Council of Supervisors of Mathematics position paper on the Ten Basic Skills Areas lists "reading, interpreting, and constructing tables, charts, and graphs" as well as "using mathematics to predict" as areas of mathematics that must be considered "basic." The National Council of Teachers of Mathematics in An Agenda for Action - Recommendations for School Mathematics of the 1980s exhorts us to place increased emphasis on:

- locating and processing quantitative information;
- collecting data;
- organizing and presenting data;
- drawing inferences and predicting from data.

There is no question but that the informed citizen must depend upon such "basic skills" in order to stay informed and make judgments on matters of importance to him/her.

Introducing young children to simple statistical procedures allows them to order and structure observations for the purpose of making sense of the world in which they live. As students progress through the grades, they will extend and refine these procedures. Students will learn to use collected data to make predictions and be introduced to simple notions of mathematical probability. Students will be provided opportunities to apply these understandings and skills in problem situations of relevance and interest to them.

As with the other strands, teachers should structure activities so that concepts are developed first at the concrete level before proceeding to the representational and abstract levels.

V-A. GRADES K-3

Learning Results	Instructional Implications
1. Students collect, organize, present and interpret data.	1.1 Teachers should provide many opportunities for students to collect information from direct observations of their environment. 1.2 Teachers should provide introductory experiences in collecting and organizing data by providing activities in which students: a. match objects to numerals in the counting sequence in a one to one correspondence; b. sort and classify subsets of objects;

Learning Results	Instructional Implications
	c. tally objects in sets and subsets; d. count objects in sets and subsets; e. list the number of objects in sets and subsets. 1.3 Teachers should provide opportunities for students to make real graphs (graphs that display the objects that have been sorted into sets to compare the associated numbers) before introducing students to picture graphs, symbolic graphs, charts, or tables. 1.4 Teachers should provide opportunities for students to work cooperatively in constructing graphs: e.g., each student places a "cake" on the birthday-month graph. 1.5 Teachers should guide students to interpret data by making comparisons and drawing conclusions from the organized data as in the following example: More students wore tie shoes than buckle shoes to school today. (simple comparison statement) or More than half the students wore tie shoes to school today. (less obvious comparison) Conclusion (subject to further verification) - Most students wear tie shoes to school.
2. Students use data to predict outcomes of future events.	2.1 Teachers should encourage students to collect and record information for the purpose of identifying similarities, differences, patterns or trends in the related samples: e.g., flipping a coin, keeping the lunch count, etc.

Bibliography of Selected Instructional Resources

Mathematics Their Way, Mary Baratta-Lorton, Addison Wesley

Probability and Statistics, Nuffield Mathematics Project, John Wiley and Sons

Other Resources

Number cubes

Spinners

Beads

Cards

Colored cubes

Familiar 2-sided objects (coins, lima beans spray-painted on one side, etc.

Sets of familiar objects such as keys, buttons, etc.

Graphing plastic for "real" graphs

V-B. GRADES 4-6

Learning Results	Instructional Implications
1. Students gather, organize, and present data in lists, tables, charts and graphs; students interpret data that is presented in lists, tables, charts and graphs.	1.1 Teachers should instruct the students in a variety of techniques in gathering data such as repeated trials, unbiased surveys, representative sampling, etc.
	1.2 Teachers should provide opportunities for students to make organized lists of collected data.
	1.3 Teachers should provide opportunities for students to present data in the form of graphs (picture, bar, line, divided bar or circle), tables, and charts.
	1.4 Teachers should provide opportunities for students to read and interpret graphs, tables, charts, and lists, and to draw inferences from the data that is displayed.
	1.5 Teachers should provide instruction and activities related to finding the mean (average), median and mode of a set of data.

Learning Results	Instructional Implications
2. Students understand the concept of probability and can find probabilities through experimentation.	2.1 Teachers should provide students with a variety of experiences leading to the understanding of the concepts of impossibility, certainty, and probability and their use in predicting the outcome of events.
	2.2 Teachers should provide activities in which students will conduct experiments to determine the probability of a given outcome.
	2.3 Teachers should encourage students to make predictions based upon experimentally obtained probabilities and to verify their predictions by collecting additional data.
3. Students determine mathematical probabilities.	3.1 Teachers should instruct students in the methods of finding mathematical probabilities in simple cases such as those involving flipping coins, rolling number cubes, etc.
	3.3 Teachers should provide opportunities for students to explore situations involving the number of subsets of a set (combinations) and the number of arrangements of objects in a set (permutations) and to use the resulting information to predict designated outcomes.
4. Students gain an understanding of the uses of probability and statistics in other disciplines, in careers, and in the real world.	4.1 Teachers should provide activities in which students use statistical methods in interdisciplinary applications: e.g., social studies, science, P.E., etc.
	4.2 Teachers should provide problem situations from a variety of careers in which probability and statistics play a part.
	4.3 Teachers should provide opportunities for students to analyze and evaluate the presentation of statistical information to the public by business, industry, the media, and government agencies.

Bibliography of Selected Instructional Resources

Statistics and Information Organizing, Mathematics Resource Project, Creative Publications

What Are My Chances, Book A, Shulte and Choate, Creative Publications

How to Lie with Statistics, Darrell Huff, W. W. Norton and Co.

The Good Time Math Event Book, Marilyn Burns, Creative Pub.

Math for Smarty Pants, Marilyn Burns, Yolla Bolly Press

Teaching Statistics and Probability, NCTM, 1981 Yearbook, National Council of Teachers of Mathematics

Mathematics, A Way of Thinking Robert Baratca-Lorton, Addison Wesley

The I Hate Mathematics Book, Marilyn Burns, Yolla Bolly Press

Probability and Statistics, Nuffield Mathematics Project, John Wiley and Sons

Other Resources

Old telephone books

Beads, thumbtacks, paper cups, etc.

Coins

Number cubes (dice)/foam, plastic or wood cubes

Sets of colored cubes

Polyhedra "dice" sets (4, 8, and 12 sided)

Spinners

Beans, beads or other counters

Blank playing cards

Chips/marbles/pebbles

Calculators

Computers

Graph paper

Learning Results	Instructional Implications
1. Students will gather, interpret, analyze, and report data.	1.1 Teachers should provide experiences in the construction and interpretation of a variety of graphs such as pictorial, bar, line, and circle or divided bar graphs.
	1.2 Teachers should provide activities that lead to the understanding and use of mean, median, mode, and range of a set of data.
	1.3 Teachers should encourage the coordination of instruction in the use of statistics and graphing across the disciplines (e.g., social studies, science, etc.)
	1.4 Teachers should emphasize the use of data that is relevant to students' interests and experience.
	1.5 Teachers should provide opportunities for students to recognize and discuss the use and misuse of statistics.
	1.6 Teachers should encourage the use of calculators and computers in the gathering, organizing and displaying of data.
	1.7 Teachers should provide opportunities for students to use a variety of methods in reporting/displaying data and in communicating interpretations and conclusions.
2. Students understand and apply concepts related to both empirical and mathematical probability.	2.1 Teachers should correctly use the vocabulary and symbolism of probability, instruct students in correct usage, and expect students to use the vocabulary and symbolism correctly.
	2.2 Teachers should provide opportunities for students to find empirical probabilities and make predictions from student generated data.
	2.3 Teachers should incorporate the study of patterns in predicting outcomes.
	2.4 Teachers should provide "real world" experiences from other disciplines

Learning Results	Instructional Implications
	as well as careers that illustrate the uses of probability.
	2.5 Teachers should provide activities that involve the use of tree diagrams, sample spaces, etc. as the means to determine mathematical probabilities.
	2.6 Teachers should provide activities in which students test empirical probabilities against the associated mathematical probabilities in cases where this is possible (e.g., probability of two heads when flipping three coins, etc.)
	2.7 Teachers should provide activities involving the use of factorials, combinations, and permutations in determining probabilities.
	2.8 Teachers should encourage students to formulate and solve problems involving probability.

Bibliography of Selected Instructional Resources

Statistics and Information Organizing, Mathematics Resource Project, Creative Publications

How to Lie with Statistics, Darrell Huff, W. W. Norton and Co.

When Are We Ever Gonna Have to Use This, Hal Saunders, Dale Seymour Publications

Teaching Probability and Statistics NCTM 1981 Yearbook, National Council of Teachers of Mathematics

USA TODAY and other newspapers

What Are My Chances? Book A, Shulte and Choate, Creative Publications

Other Resources

Graph paper

Protractor, compass, ruler

Paper cups, thumbtacks, etc. for probability experiments

Sets of colored cubes, chips or other counters

Polyhedra "dice" (4, 6, 8, and 12 sided)

Spinners

Blank playing cards

Calculators and computers

VI. K-8 ALGEBRA STRAND

Students are introduced to "open sentences," in which boxes or blanks serve as variables, at a very early age. This sometimes occurs in the context of finding missing addends or missing factors. As with all other instruction in concept development, there should be a progression from the concrete, through the pictorial, to the abstract level (The common difficulty with missing addends and factors at the primary level most often occurs because not enough experiences are afforded at the concrete level.) Many advise that the beginning experiences at the concrete level should be entirely verbal with "story telling" taking the place of pencil/paper work. With young students, the teacher should act as the "scribe" and record the mathematical "action" in symbolic form on the chalkboard or overhead projector so that the students make the proper association between their work with manipulatives and the symbolic form. As students progress through the grades, they learn to generate and solve open sentences at the abstract level, but always with concrete and pictorial representations available for reference as needed. When introductory algebra skills are taught, elementary students should understand that they are "doing algebra" which will serve to reduce any anxieties that they may have as they approach the formal study of this area of mathematics in future years.

Basic algebraic concepts and skills as developed through grade 8 are needed by all students whether in preparation for the vocations that they may eventually choose, for the solution of "real life" problems, or for successful entry into a formal study of algebra and subsequent higher-level mathematics courses at the high school level.

VI-A. GRADES K-3

Learning Results	Instructional Implications
1. Students use manipulatives and pictorial representations to generate and solve simple open sentences.	1.1 Teachers should provide a great many experiences in which manipulatives are used to generate and solve open sentences with the teacher acting as "scribe" until such time as students can record the results of their investigations for themselves.

Bibliography of Selected Instructional Resources

CDA Mathematics materials:
Drill and Practice at the Problem Solving Level
Banking on Problem Solving
Robert W. Wirtz, Curriculum Development Associates

Other Resources

Manipulative materials such as beans, blocks, etc.

Learning Results	Instructional Implications
1. Students plot ordered pairs of numbers on a grid and "read" points on a grid as ordered pairs of numbers.	1.1 Teachers should provide instruction in matching ordered pairs of whole numbers with points on a grid.
	1.2 Teachers should provide practical applications associated with 1.1 above such as using the coordinates given on maps to locate towns, etc.
2. Students generate and solve open mathematical sentences.	2.1 Teachers should provide experiences that lead students to the understanding that the "=" symbol does not imply that an "answer" is about to follow; rather, an equation is a balance of quantities.
	2.2 Teachers should utilize a variety of word problems in helping students learn to state in symbolic form the mathematical sentences needed to solve the problem; and provide an abundance of such problems that require students to generate and solve open sentences (as opposed to isolated drill in which the student must just write the "answers").
	2.3 Teachers should treat the use of formulas (such as $P = 2\ell + 2w$) within the context of open sentences.
	2.4 Teachers should encourage students to use calculators in solving open sentences whenever the emphasis is on the method of solution and not on the computational processes.

Bibliography of Selected Instructional Resources

Mathematics, A Way of Thinking, Robert Baratta-Lorton, Addison Wesley

Graphs Leading to Algebra, Nuffield Mathematics Project, John Wiley and Sons

The Good Times Math Event Book, Marilyn Burns, Creative Publications

Point Counterpoint, Ken Rand, Creative Publications

Other Resources

Geoboards, dot and grid paper	Variety of "real life" word problems
Number balance scales	Calculators

Learning Results	Instructional Implications
1. Students use variables in open sentences, evaluate expressions, solve simple linear equations, and perform basic operations with integers.	1.1 Teachers should use correctly the vocabulary and symbolism of algebra, instruct the students in correct usage, and expect them to understand and use the vocabulary and symbolism correctly.
	1.2 Teachers should provide activities that require students to use the rules for order of operations, exponents, and properties of operations in simplifying expressions.
	1.3 Teachers should provide opportunities for students to translate word phrases and sentences into mathematical expressions and equations; and vice versa.
	1.4 Teachers should provide instruction in solving linear equations together with opportunities for students to exercise the associated skills.
	1.5 Teachers should provide experiences that lead to an understanding of ratio and proportion and opportunities for using these understandings in solving problems.
	1.6 Teachers should provide a variety of word problems at the appropriate level and guide the student toward the development of algebraic methods of solution.
	1.7 Teachers should make use of concrete experiences and models in developing integer concepts and provide opportunities for students to practice the basic operations with integers.
2. Students develop understandings associated with linear equations in two variables and can graph solutions of linear equations in the coordinate plane.	2.1 Teachers should use "guess my rule" activities to develop an understanding of the solution of a linear equation in two variables as a set of ordered pairs.
	2.2 Teachers should present activities that provide students practice in graphing solution sets of linear equations in two variables.

Bibliography of Selected Instructional Resources

Pre-Algebra with Pizzazz, Steve and Janice Marcy, Creative Publications

When Are We Ever Gonna Have to Use This?, Hal Saunders, Dale Seymour Publications

Other Resources

Graph paper

Number lines

Thermometers

Calculators

Games (Battleship, etc.)

Guess My Rule activities

Generating magic squares from magic squares activities

Bibliography of General Interest Publications

Math for Girls and Other Problem Solvers, Downie and others, Lawrence Hall of Science, University of California

The Fabric of Mathematics, Laycock and Watson, Activity Resources Co.

Mathematics, A Human Endeavor (for grades 7-8), Harold Jacobs, W. H. Freeman and Co.

NCTM Yearbooks:
- The Agenda in Action (1983)
- Mathematics for the Middle Grades (1982)
- Teaching Statistics and Probability (1981)
- Problem Solving in School Mathematics (1980)

National Council of Teachers of Mathematics

Mathematics Library - Elementary and Junior High School, Wheeler and Hardgrove, National Council of Teachers of Mathematics

The Arithmetic Teacher magazine, National Council of Teachers of Mathematics

The Mathematics Teacher magazine, National Council of Teachers of Mathematics

Multicultural Posters and Poster Activities, WSMC, 1982, Single Topic Edition, National Council of Teachers of Mathematics

Other WSMC Single Topic Editions:
- Make It and Take It Math Games and Activities (1980)
- Developing Thoughtful Behavior in Mathematics (1983)
- Using Calculators to Develop Thoughtful Behavior in Mathematics (due Spring, 1984)

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